Impact of Killing vector fields on Riemannian and spacelike hypersurfaces

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Let $\left(\overline{M},\overline{g}\right)$ be an $(n+1)$-dimensional either Riemannian or Lorentzian manifold, and let $\overline{\xi}\in\mathfrak{X}(\overline{M})$ be an arbitrary vector field that we assume to be timelike in the case where $\overline{M}$ is Lorentzian. Let (M,g) be a connected n-dimensional Riemannian manifold that is isometrically immersed as a hypersurface into $\left(\overline{M},\overline{g}\right)$, and let ξ denote the restriction of $\overline{\xi}$ to M.

Our main goal in this talk is to give a useful expression for the Laplacian $\Delta\theta$ of the function $\theta=\overline{g}\left(\xi,N\right)$, where $\overline{\xi}$ is an arbitrary vector field and N is a globally defined unit vector field normal to M. In the case where $\left(\overline{M},\overline{g}\right)$ is Riemannian and $\overline{\xi}$ is a conformal Killing vector field, we meet an expression for $\Delta\theta$ that has been given in literature in terms of the Ricci curvature of $\left(\overline{M},\overline{g}\right)$ and the norm of the shape operator. We also derive some interesting results concerning the impact of the existence of Killing vector fields on Riemannian and spacelike hypersurfaces.

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